Stress Analysis

Mechanics: Stress and strain

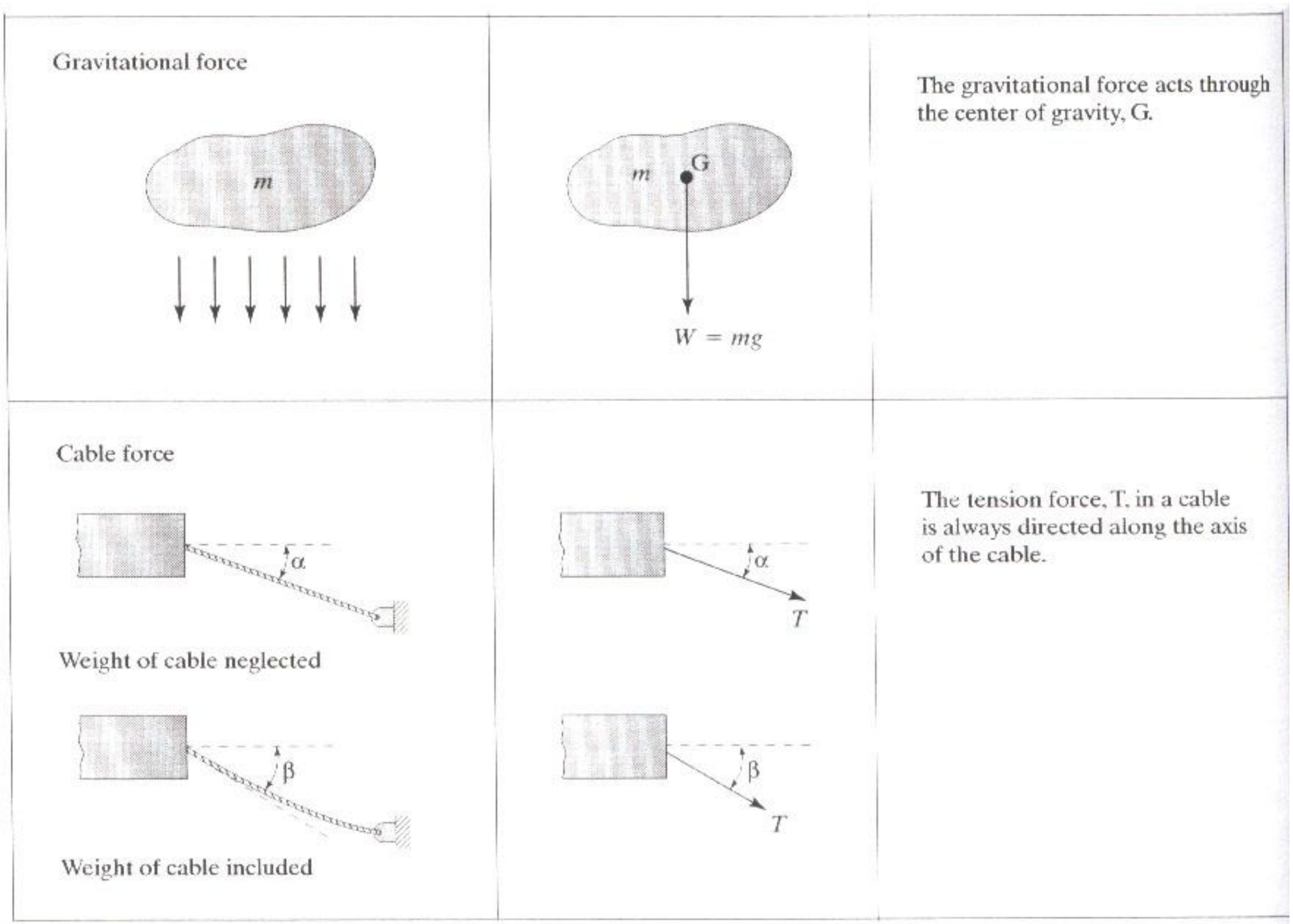
Topics

- Free Body Diagrams (Review)
- Stress
- Strain
- Deformation
- Hooke’s Law
- Stress-Strain Diagrams
- Design: The Safety Factor

Free Body Diagrams

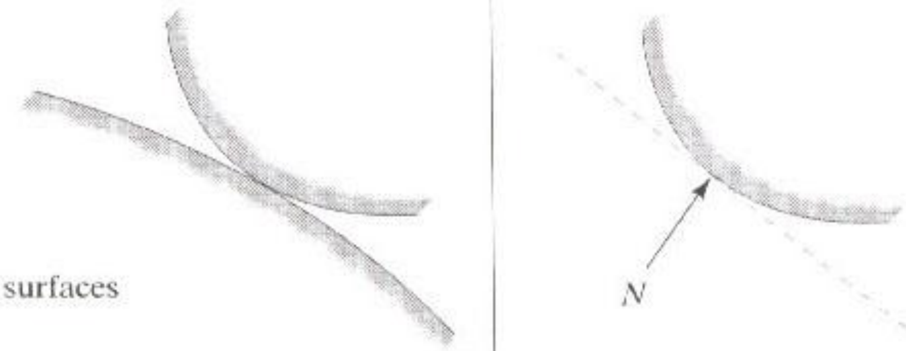
- Shows all external forces acting on the body
- Procedure
  1. Identify the isolated body and draw it
  2. At all force locations draw the vectors
     a. Supports
     b. Connections
     c. Contacts
  3. Add the Weight force
  4. Label forces with their value or a letter for unknowns
  5. Add a coordinate system
  6. Add geometric data (Lengths, angles, …)

Examples 1


Gravitational force
m
m
G
W = mg
The gravitational force acts through the center of gravity, G.
Cable force
α
Weight of cable neglected
β
Weight of cable included
α
T
β
T
The tension force, T, in a cable is always directed along the axis of the cable.

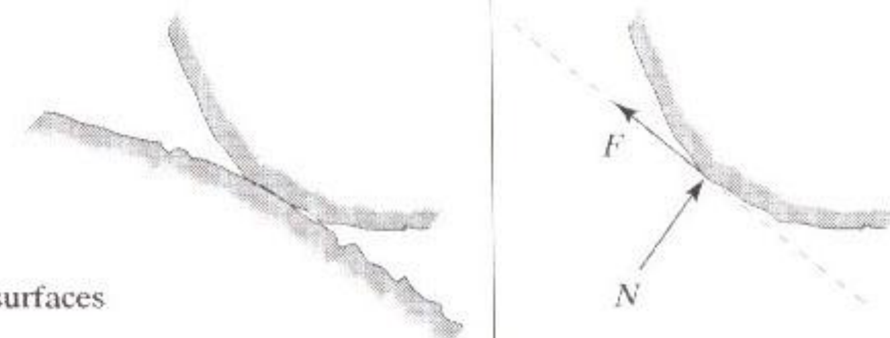
Examples 2

Contact force



Smooth surfaces

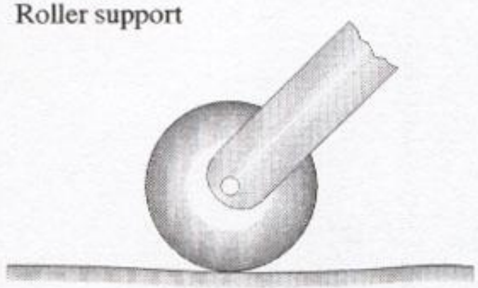
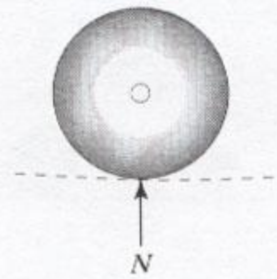
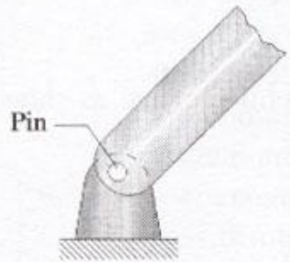
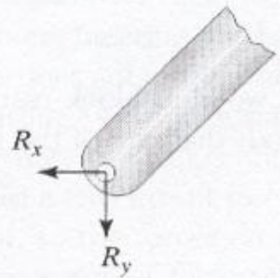
For smooth surfaces, the contact force, N, is toward the body, normal to the tangent drawn through the point of contact.



Rough surfaces

For rough surfaces, there are two forces, a normal force, N, and a friction force, F. These two forces are perpendicular to each other. The friction force, F, acts in the direction opposite of the impeding motion.

Examples 3

Configuration	Free-body diagram	Comments
Roller support 		A roller supports a normal force but no friction force because a friction force would cause the roller to rotate.
Pin connection 		A pin connection can support a reaction force in any direction in the plane normal to the pin's axis. This force may be resolved into its x and y components, R_x and R_y.

Stress

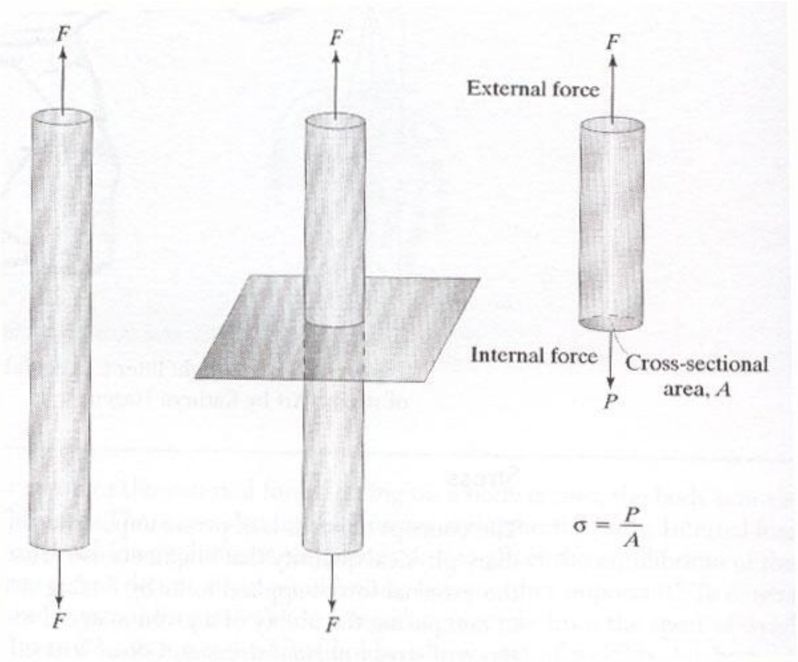
- Internal Forces
  - Reaction to external forces
  - Distributed throughout volume of material
- Stress
  - Normal stress
    - Acts at right angles to a selected plane
    - The axial direction in a rod or cable
  - Shear stress: stays in the selected plane (advanced topic: Strength of materials course)
  - **Can a structure withstand the forces applied to it?**

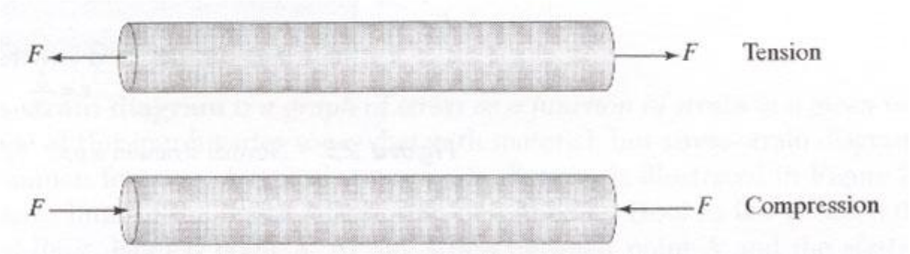
Stress 2

$\sigma = P/A$ (Pascals)

- σ: Average stress
- P: Applied force
- A: Cross section area

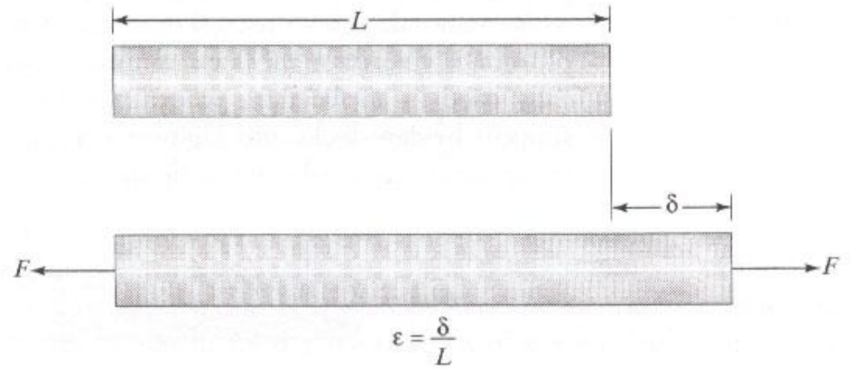
Tension / Compression





Strain

- Strain: deformation due to stress
  - $\varepsilon = \delta / L$
    - ε: Normal strain
    - δ: Change in length
    - L: Original (unstressed) length



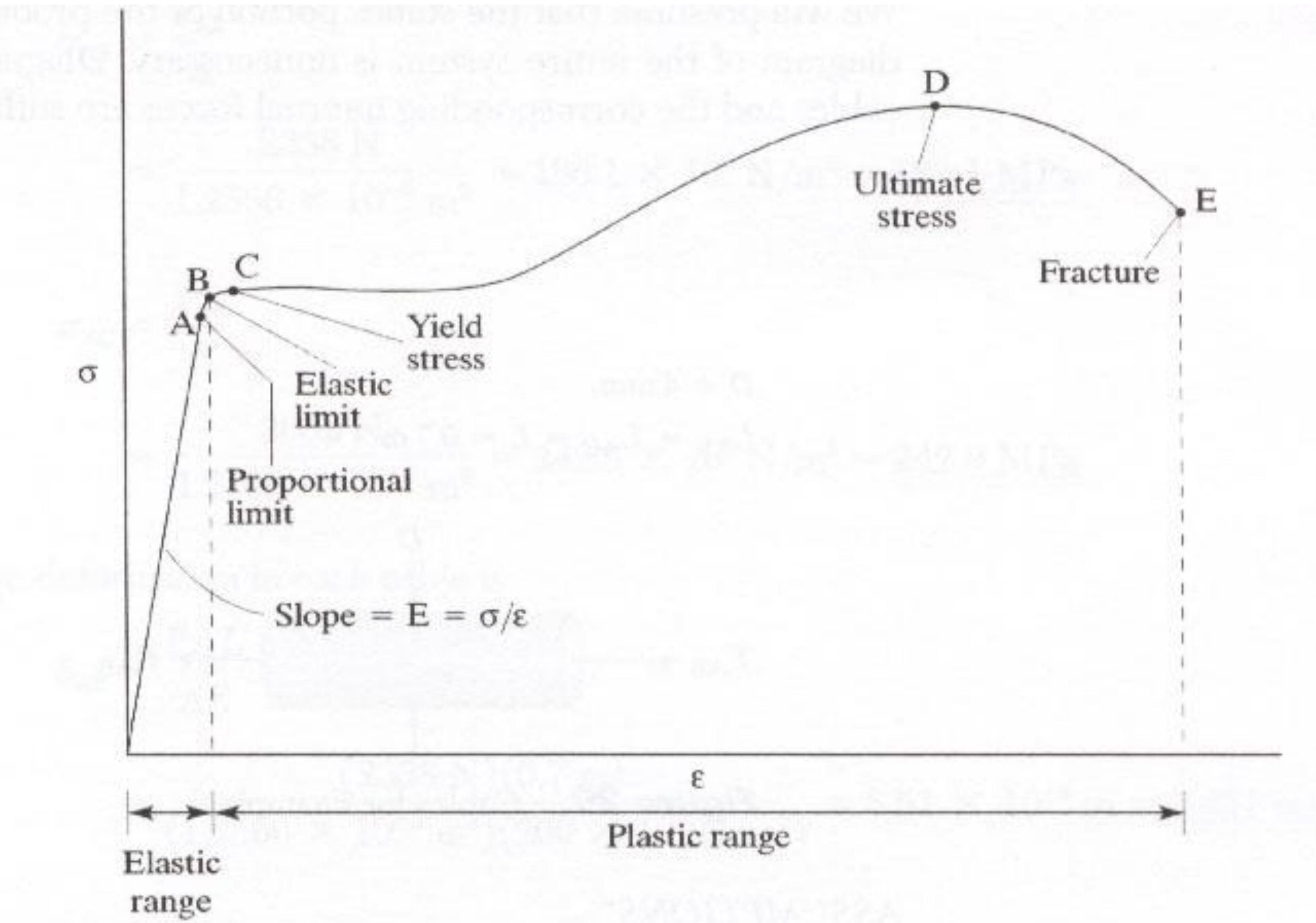
Hooke's Law

- Springs
  $F = kx$
  F: Applied Force
  k: Spring constant
  x: displacement
- Stress-Strain
  $\sigma = E\varepsilon$
  σ: Stress
  E: Modulus of Elasticity
  ε : Strain

- Using
  $\sigma = P / A$ and
  $\varepsilon = \delta / L$

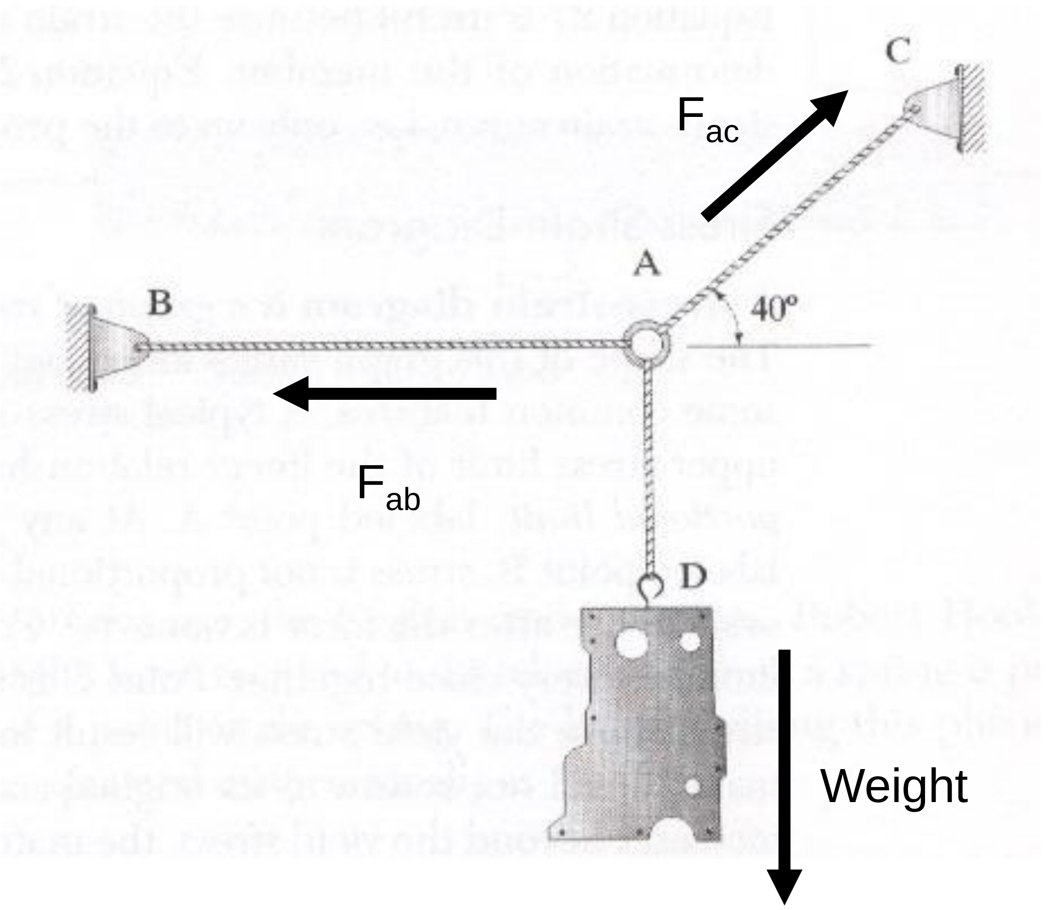
$$\delta = \frac{P * L}{A * E}$$

Stress-Strain Diagram



Example 6

- 200 kg engine block
- Cables
  - 0.7 m long
  - 4 mm diameter
  - E = 200 GPa
- Find
  - Normal Stress
  - Axial Deformation



F = 200 kg * 9.8 m/sec^2

F = 1960 newtons